

Developing a ‘Conjecturing Atmosphere’ in the Classroom through Task Design and Enactment

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In recent years there has been an increased emphasis on algebraic reasoning in primary school classrooms. This includes introducing students to the mathematical practices of making conjectures, justifying and generalising. Drawing on findings from a classroom-based study, this paper explores one teacher’s journey in shifting her task design and enactment to develop a ‘conjecturing atmosphere’ in the classroom. The findings affirm the important role of the teacher in introducing mathematical practices. Careful task design and enactment, teacher questioning, and noticing and responding to student reasoning were important elements in facilitating conjecturing, justifying and generalising.

Important changes have been proposed for mathematics classrooms in recent years in response to a consideration of how mathematics education can best meet the needs of students in the 21st century and support them to participate in our developing ‘knowledge society’. One aspect of the proposed changes is an increased emphasis on algebraic reasoning in primary school classrooms (Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003). Task design and enactment, along with pedagogical actions, are important factors in developing early algebraic reasoning in classrooms. When using existing curriculum material teachers need to recognise and adapt material to exemplify opportunities and enact planned tasks in such a way that algebraic reasoning occurs. In addition, they need to recognise spontaneous opportunities for algebraic reasoning while the tasks are enacted and use pedagogical actions to facilitate these opportunities.

Importantly, in classrooms where frequent viable algebraic reasoning opportunities occur students make purposeful conjectures, construct mathematical arguments, justify, and generalise their ideas (Blanton & Kaput, 2005). While clearly this is a critical component if students are to access algebraic reasoning, this often is not achieved. This is because the mathematical practices inherent in task designs and enactment are complex and challenging for teachers. The research reported in this paper provides an exemplar of one teacher’s journey in shifting her task design and enactment practices to provide opportunities for her students to engage in mathematical practices aligned with developing algebraic reasoning.

Making conjectures and developing these into generalisations along with justifying mathematical ideas and developing age appropriate proof are key mathematical practices linked to the development of early algebraic reasoning (Bastable & Schifter, 2008). However, as Mason (2008) maintains too often it is the teacher who provides the examples, cases, and methods during mathematics—a practice which constrains the space for students to generalise. He argues for the need to develop a “conjecturing atmosphere” in the classroom where the expectation is that generalisations will be expressed and treated as conjectures and then justified. Creating a classroom culture which focuses on generalisation and justification is not an easy task as many teachers themselves may not have had experience in constructing and justifying generalisations or promoting these

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practices in their classrooms. There are also considerable challenges related to the difficulties students may encounter in both constructing and justifying generalisations. These challenges have been attributed to a lack of understanding of generality along with difficulties with mathematical language and symbolism and a lack of problem-solving skills necessary to construct an argument (e.g., Anthony & Walshaw, 2002; Callingham, Falle, & Clark, 2004; Chick, 2009)

The teacher takes an important role in facilitating the development of a conjecturing atmosphere in the classroom by both planning opportunities for generalisation and drawing on spontaneous opportunities during task enactment. In planning opportunities for generalisation tasks may be purposely designed to elicit conjectures from students. For example, Carpenter, Levi, Franke, and Zeringue (2005) describe how true and false number sentences can be used to draw student attention to the distributive property of multiplication. Alternatively, teachers can draw on spontaneous opportunities by carefully monitoring student observations and questions during small group work and identifying student generated conjectures for exploration. For example, Schifter, Monk, Russell, and Bastable (2008) report on a classroom episode where young students were working to generate ways to make ten. As the teacher observed the students working, she noted that many of them were utilising the commutative principle which they had informally termed 'turn arounds'. Noting that some students were making statements such as: *turn arounds always work* prompted the teacher to use this to lead a discussion and probe understanding of additive commutativity.

Other pedagogical actions are also important during task enactment to facilitate student engagement in mathematical practices. For example, teacher questioning may be used to focus student attention on the patterns and relationships within the task (Smith & Thompson, 2008). Following the development of conjectures, students need to engage in testing the conjectures to investigate whether they are true. This requires the teacher to position students to agree or disagree based on mathematical arguments and to facilitate students to use concrete materials and representations to develop their arguments. For example, Bastable and Schifter (2008) provide an example of a teacher positioning students and facilitating them to draw on visual representations to further develop and explore the generalisations they had constructed about square numbers.

The theoretical framing of this paper is based within a socio-cultural perspective. In this view, individuals participate in the everyday activities within a classroom community of practice (Lave & Wenger, 1991) and through this participation learn the ways of thinking and acting which are valued by the community. This facilitates the development of a sense of what it means to be a member of a specific community and supports members to increasingly participate in more sophisticated ways in the set of collective practices which are valued.

Methodology

This paper reports on episodes drawn from a larger study which involved a year-long professional development classroom-based intervention focused on developing early algebraic reasoning. The participants included two separate groups of primary teachers (one group from England, the other from the British Isles) from schools which used the Mathematics Enhancement Programme (MEP) curriculum material. This curriculum material includes resources such as lesson plans, workbooks, and online interactive

resources. Many of the tasks in the curriculum material have implicit opportunities to facilitate students to engage in algebraic reasoning due to their structural basis.

The focus in this paper is on one teacher and her 25 Year Three students from a semi-rural primary school in the British Isles. The students were from predominantly middle socio-economic home environments and represented a range of ethnic backgrounds. The teacher was an experienced teacher who was interested in strengthening her ability to develop early algebraic reasoning within her classroom.

An initial model for professional development was designed based on research literature. The subsequent re-design drew on a range of sources including researcher observations from the classrooms, study group meetings, teacher interviews and discussions. For example, from the classroom observations and discussions during study group meetings, the researcher observed that the teachers needed professional development in facilitating students to generate and explore conjectures. In response a task was designed to enable the teachers to explore possible conjectures which students would make and how these could be justified.

Another central focus for the professional development was the selection, design, and enactment of tasks. Drawing on previous studies (e.g., Blanton & Kaput, 2008; Franke, Carpenter, & Battey, 2008; Schifter et al., 2008) an aim was to use algebraic tasks to provide the teachers with multiple opportunities to reflect on their own and their students' understanding of algebraic concepts and the mathematical practices which support students' learning of early algebra. For example, the teachers were asked to solve number sentences involving variables, develop their own number sentences, and at another time asked to develop different forms of justification for a conjecture. Drawing on tasks from the MEP curriculum material teachers were encouraged to identify opportunities for algebraic reasoning and also to investigate ways of modifying and further developing existing tasks.

Data gathering included classroom observations prior to and during the year-long professional development, video records of professional development meetings and, audio recorded interviews, detailed field notes, and classroom artefacts. On-going data analysis supported the revision of the professional development. Retrospective data analysis used QSR International's NVivo 10 qualitative software programme (2012). This included multi-levels of coding using both parent and child nodes. The initial codes were developed from a variety of sources including research literature, the initial viewing of the video records, and the observational and reflective field notes. Repeated viewing of the videos and re-reading of the transcripts and field notes confirmed or refuted the initial hypotheses and codes and other hypotheses and codes were developed as necessary.

Findings

Within this section an exemplar is provided of a teacher's journey from using and enacting tasks in a teacher directed, procedural way to using tasks to engage students in making conjectures, justifying these and developing generalisations in rich and meaningful ways. This is presented through examples of episodes observed in the classroom during different phases of the study.

Prior to the Professional Development

Prior to the professional development the teacher used and enacted tasks in a teacher directed, procedural way. Opportunities to explicitly identify or examine the properties of numbers and operations were not drawn upon. For example, in one lesson the students constructed two alternative solutions that implicitly drew on the commutative property. The teacher began by recording these on the whiteboard and asking the students to describe what they had noticed before offering a brief explanation of the commutative property herself:

Otto: It's the other way around...it's, it's the same but it's just changed around

Mrs Stuart: And that's one of the really important things in multiplication, isn't it? It doesn't matter if we do two times five or five times two.

This limited the opportunities for students to develop deep, generalised understanding as advocated by many researchers (e.g., Anthony & Walshaw, 2002; Carpenter et al., 2003; Schifter et al., 2008).

Phase One: Early Changes to Task Design and Enactment

Following the initial professional development meeting, the teacher began intentionally developing and trialling ways of adapting her planning to focus student attention explicitly on algebraic concepts. She examined the MEP lesson plans and rather than asking students to complete the whole task, she presented them with parts of the task which focused their attention on the properties of numbers or operations. In one lesson the teacher planned to use a task involving an array and two number sentences with missing parts (e.g., $3 \times _ = 6$, $6 \div _ = 2$) to focus student attention on the inverse relationship between multiplication and division. As we see below, initial teacher questioning was used to focus student attention on the general relationship between multiplication and division:

Teacher (records $3 \times 2 = 6$ and $6 \div 3 = 2$) Let's have a look at those, did anyone notice anything? Three times two equals six and six divided by three equals two. With your partner, what do you notice about those please?

After the students talked with their partners, she asks a student to say what he noticed:

Tristan They're just the other way around... because the three is in the middle and the six is at the beginning and at the end.

The teacher then directed the students to examine related equations where the position of the numerals has changed. However this shift in focus and the following teacher questioning moved the focus to specific equations limiting the opportunities for students to further explore the relationship between multiplication and division.

Teacher: So it's the same digits. Would it work if I put them in any order? If I did this (*writes $2 \div 3 = 6$ on the board*) two divided by three equals six because I've got the same numbers. Just talk that one through with your partner or what about this one, three divided by six equals two, is that true? Or six divided by three equals two (*writes the different equations on the board*) Are any of those true?

This was followed by further whole class discussion involving individual students using magnetic counters to model whether each equation was true. By asking the students to use magnetic counters to solve each equation, their attention was shifted specifically to calculating answers and thus the focus on the inverse relationship was lost. In this case

concrete material was introduced as a tool to solve the task rather than as a means of developing an argument and proving or justifying. The lesson concluded with the teacher writing the equation $a \times b = c$ and then stating a conjecture:

Teacher: I have this theory that for every pair of factors and a product I can make two multiplications and two divisions let's see if that's right. With your partner at Planet X can you see if you can come up with equations for that?

Opportunities for the students to develop and explore their own conjectures and prove and justify their reasoning were missed by the teacher telling the students the conjecture that she had developed and then guiding them towards simply generating equations to match the conjecture.

Thus at this stage of the professional development, although the teacher had begun to plan for algebraic reasoning there were still limited opportunities for engagement with mathematical practices associated with algebraic reasoning. For example, key mathematical practices such as making conjectures, developing generalisations, justification and proof (Bastable & Schifter, 2008; Carpenter et al., 2003; Mason, 2008) were not established within the classroom during this phase. Mrs Stuart's practice of seeking examples and cases was promising, but her propensity to offer conjectures potentially reduced student opportunity to generalise (Mason, 2008).

Phase Two: Shifts in Task Design and Enactment

In the second phase of the study, the teacher drew on research and case studies (e.g., Carpenter et al., 2003; Schifter, 2009) presented during a professional development meeting to introduce the mathematical practices of generalisation, justification, and proof. She purposefully planned an investigation of zero with the aim of students developing conjectures, followed by justification and generalisations of their thinking. In this lesson student attention was drawn to a number sentence which had been constructed to reach the target number of 20 (e.g., $20 + 0 = 20$) and they were asked to discuss what they noticed. The teacher then facilitated the students to develop a conjecture and find examples which illustrated the conjecture. Following this, she pressed them beyond the use of examples as justification by requiring that they prove their conjectures using a range of concrete materials (e.g., acting out the scenario and using counters) before she asked them to symbolise it. Similar to the finding of Carpenter and his colleagues (2003), this exploration of the properties of zero was a rich area to scaffold students to develop and investigate conjectures and generalisations. The context also provided the students with opportunities to use concrete materials and representations as a means to develop an argument and establish a general claim.

Although the teacher had now begun to adapt her task design to include opportunities to engage students in mathematical practices at this point in the study this did not extend to drawing on spontaneous opportunities during task enactment. For example, when the students referred to odd and even numbers or other patterns they had noticed, she heard them but did not develop them further. In one lesson a student listened to two solution strategies and noted:

Julio: It's the commutative law like it's just the other way around.

Later the teacher commented on this:

Teacher: I was really impressed that they retained things from last term. You know Julio was like 'oh that's the commutative law'.

Although she noticed the statement, on this occasion she did not use the opportunity to engage students spontaneously in further investigation.

In subsequent observations during the final part of this phase of the study, the teacher began to recognise and draw upon spontaneous opportunities within enacted mathematical tasks. This included student generated conjectures about the patterns they noticed. The teacher used these as opportunities to engage students in the mathematical practices of generalisation, justification and proof. In one instance the teacher asked the students to use 12 counters and write number sentences related to these. She recorded two related number sentences ($\frac{1}{2}$ of 12 = 6 and $12 \div 2 = 6$) on the board and asked the students what they noticed. A student (Paul) made a conjecture that to find a half you can divide by two. After recording this initial conjecture the teacher then asked her students to work in pairs to investigate Paul's conjecture by exploring what happened when 12 counters were divided by two. She then asked them to extend this into different fractional parts including thirds and quarters. The whole class discussion began with a student agreeing with the conjecture Paul had made:

Jasia: It is because one third is three and there is three here and you have divided them all by the same so the same as 12 and 12 divided by three equals four.

The teacher revoiced the explanation and then asked the students to generalise the conjecture:

Teacher: Paul wanted to know, his idea was: is dividing by two the same as finding a half. This time we've divided by three is that the same as finding a third? Jasia agreed with that and coming back to Paul's idea, dividing by two is the same as finding a half. Can anyone think what dividing by n would be the same as?

After further discussion the teacher returned to Paul who had made the original conjecture:

Paul: Finding an n th.

In this phase of the study the teacher continued to recognise opportunities within the curricular material which could be extended through task design to facilitate mathematical practices. This was then extended to noticing and using spontaneous opportunities within enacted tasks. The teacher initiated a growing expectation that generalisations would be expressed and treated as conjectures. In doing this, she was beginning to facilitate a 'conjecturing atmosphere' such as described by Mason (2008).

Phase Three: Development of a 'Conjecturing Atmosphere'

The teacher now designed tasks and carefully considered how to enact them in such a way that exemplified opportunities for students to engage in mathematical practices. She described herself thinking about opportunities as she adapted the task design:

Teacher: Draw out the commutative law from this one, or this could be a great discussion point for, like the other week when we were doing timesing by one, or dividing by zero, get them to come out with conjectures.

She was able to identify the significant shifts the students had made in making conjectures and generalising and the key role she took in developing these practices:

Teacher: They come up with conjectures, but if they weren't asked the same sort of questions, you know if the language of conjecture and generalisation suddenly stops then that's

going to filter further away from them and I want them to be able to build on what they've got.

During the lessons the teacher maintained the expectation that conjectures would be expressed and proved while facilitating a consistent expectation for generalisation. She used questioning such as: *Would it work for different numbers?* and *Can I change that into something that would work for any number?*

In this final phase, the teacher increasingly introduced representations as a way of providing concrete justification for conjectures and generalisations. To do this the earlier norms which had been developed were built upon and an expectation was established that the students would justify their conjectures by using concrete material. For example, a student made a conjecture about dividing by one:

James: It's just like you're getting one group and dividing it by one group so you have already done it. If you've got a number and you divide it by one, it ends up that number.

Teacher: Show what you mean with counters on the board.

As the students gained more experience in justification, they more readily drew on material to prove their reasoning. In one lesson which focused on a task involving the distributive property (e.g., Write the correct sign for $9 \times 14 _ 9 \times 7 + 9 \times 7$), the teacher facilitated the students to draw on representations to justify their reasoning. Building on previous work which investigated how relational reasoning could be used to solve tasks involving the distributive property, many students began to generalise the distributive property to solve the tasks. The teacher asks a student to share her explanation:

Misty: Seven add seven is 14 [*notates an arrow from each seven and writes 14 underneath*] and there is a 14 there [*indicates 14 on the left-hand side*] and they are both times nine so you have got 14 times nine and 14 times nine.

The teacher then asks the students to work in pairs using Misty's reasoning to prove whether $9 \times 6 = 9 \times 3 + 9 \times 3$. A student begins by building an array to represent 9×6 , Misty then develops this further:

Misty: Because there is three there [*indicates splitting the six rows into three by drawing a line*]. There is three rows there and three rows there and that is just the same as those [*points to $3 \times 9 + 3 \times 9$ in the equation*] and then it is times nine [*points across the rows*].

In the final phase of the project the teacher consistently engaged her students in building generalisations in the classroom. She achieved this through noting the conjectures that students made and then facilitating the whole class to investigate these. This involved testing and revising the conjecture and developing it into a generalisation. Building on the new expectation that students would justify their conjectures using concrete materials students began to use representations to develop reasoned, general arguments. The result of these actions was a 'conjecturing atmosphere' being developed in the classroom.

Conclusions

This study sought to illustrate the pathway which a teacher took in shifting her task design and enactment from a teacher directed, procedural way to designing and enacting tasks in such a way that a 'conjecturing atmosphere' was developed in the classroom. Similar to the findings of other researchers (e.g., Carpenter et al., 2005; Schifter et al., 2008; Smith & Thompson, 2008) the teacher took an important role in facilitating the

development of mathematical practices and specific pedagogical actions were required. These included the use of questioning, noticing and responding to student reasoning and pressing students to develop arguments.

In the initial phase although the teacher was beginning to plan for algebraic reasoning, the classroom practices were similar to those described by Mason (2008) whereby the teacher provided the conjectures. This constrained the space for students to generalise. The first significant shift involved the teacher purposefully designing a task which introduced her students to the mathematical practices of making conjectures, justifying and generalising. Further shifts saw the teacher move beyond planned opportunities to recognise and draw upon spontaneous opportunities within enacted mathematical tasks. The teacher began to readily draw on conjectures she heard her students making and representations were introduced as a powerful form of concrete justification.

Overall the focus on developing a conjecturing atmosphere in the classroom increased student participation in mathematical practices as they more readily made conjectures about patterns which they noticed and drew on material to support and prove their reasoning.

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